Worcester County Mathematics League

Freshman Meet 1

October 24, 2018

COACHES' COPY

ROUNDS, ANSWERS, AND SOLUTIONS

Freshman Meet 1 – October 24th 2018

Round 1: Order of Operations

All answers must be in simplest exact form in the answer section

NO CALCULATOR ALLOWED

1. Follow the proper order of operations and evaluate:

$$5[7 + 35 \div 5 * 2^2 - 3(7 + 4^2 - 12)]^2$$

2. Follow the proper order of operations and evaluate:

$$(5 \div 6 - 2^3 \div 21) \div (2 * 11 \div 14 + 22 \div 15)$$

3. Let $x \ @ \ y = \frac{x^2}{\sqrt{y}}$ and $x \ \# \ y = \frac{-x}{-y}$

   Find the simplified value of:

$$\left(\left(3 \ @ \ \frac{1}{4} \right) \ \#6 \right) \# \frac{1}{2}$$

ANSWERS

(1 pt.) 1. _____

(2 pt.) 2. _____

(3 pt.) 3. _____

Freshman Meet 1 – SOLUTIONS

Round 1: Order of Operations

1. Follow the proper order of operations and evaluate:

$$5[7 + 35 \div 5 * 2^2 - 3(7 + 4^2 - 12)]^2$$

Solution:

$$5[7 \div 5 * 4 - 3(7 + 16 - 12)]^2$$

$$5[7 + 35 \div 5 * 4 - 3(11)]^2$$

$$5[7 + 28 - 33]^2$$

$$5(2)^2$$

$$5 * 4$$

$$\underline{20}$$

2. Follow the proper order of operations and evaluate:

$$(5 \div 6 - 2^3 \div 21) \div (2 * 11 \div 14 + 22 \div 15)$$

Solution:

$$(5 \div 6 - 2^3 \div 21) \div (2 * 11 \div 14 + 22 \div 15)$$

$$(\frac{5}{6} - \frac{8}{21}) \div (\frac{22}{14} + \frac{22}{15})$$

$$\frac{\frac{35}{42} - \frac{16}{42}}{22 * (\frac{1}{14} + \frac{1}{15})}$$

$$\frac{\frac{19}{42}}{22 * (\frac{14 + 15}{14 * 15})}$$

$$\frac{\frac{19}{2 * 3 * 7}}{\frac{11 * 19}{7 * 3 * 5}}$$

$$\frac{19}{2*3*7} * \frac{7*3*5}{11*19}$$

$$\frac{5}{2*11}$$

$$\frac{5}{22} \text{ or } 0.2\overline{27}$$

3. Let $x @ y = \frac{x^2}{\sqrt{y}}$ and $x \# y = \frac{-x}{-y}$

Find the simplified value of:

$$\left(\left(3 @ \frac{1}{4} \right) \# 6 \right) \# \frac{1}{2}$$

Solution:

$$3 @ \frac{1}{4} = \frac{3^2}{\sqrt{\frac{1}{4}}} = \frac{9}{\frac{1}{2}} = 18$$

$$18 \# 6 = \frac{-18}{-6} = 3$$

$$3 \# \frac{1}{2} = \frac{-3}{-\frac{1}{2}} = 6$$

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Round 2: Linear Equations

All answers must be in simplest exact form in the answer section

NO CALCULATOR ALLOWED

1. Solve for x:

$$\frac{1}{2}(4x - 20) + 3x = 2(2x - 1)$$

2. Solve for x:

$$\frac{3}{8}x - 5.2 = x - 2\frac{3}{5}$$

3. $ax + by = 36$ and $bx - ay = -8$ intersect at $(2, 6)$

   What is the value of $|a + b|$?

ANSWERS

(1 pt.) 1. _____

(2 pt.) 2. _____

(3 pt.) 3. _____

Freshman Meet 1 – SOLUTIONS

Round 2: Linear Equations

1. Solve for x:

$$\frac{1}{2}(4x - 20) + 3x = 2(2x - 1)$$

Solution:

$$\frac{1}{2}(4x - 20) + 3x = 2(2x - 1)$$

$$2x - 10 + 3x = 4x - 2$$

$$2x + 3x - 4x = 10 - 2$$

$$x = 8$$

2. Solve for x:

$$\frac{3}{8}x - 5.2 = x - 2\frac{3}{5}$$

Solution 1:

$$\frac{3}{8}x - 5.2 = x - 2\frac{3}{5}$$

$$\frac{3}{8}x - \frac{26}{5} = x - \frac{13}{5}$$

$$-\frac{13}{5} = \frac{5}{8}x$$

$$-\frac{104}{25} = x$$

$$\text{or } -4\frac{4}{25}$$

Solution 2:

$$\frac{3}{8}x - 5.2 = x - 2\frac{3}{5}$$

$$0.375x - 5.2 = x - 2.6$$

$$-2.6 = 0.625x$$

$$-4.16 = x$$

3. $ax + by = 36$ and $bx - ay = -8$ intersect at $(2, 6)$

   What is the value of $|a + b|$?

Solution:

$$2a + 6b = 36$$
$$2b - 6a = -8$$

$$a = 18 - 3b$$

$$2b - 6(18 - 3b) = -8$$
$$2b - 108 + 18b = -8$$
$$20b = 100$$

$$b = 5$$

Substitute and solve for a...

$$2a + 30 = 36$$

$$a = 3$$

$$|a + b| = |3 + 5| = 8$$

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Round 3: Logic Problems

All answers must be in simplest exact form in the answer section

NO CALCULATOR ALLOWED

1. One jump is two hops. One hop is four skips. How many skips are in a hop, skip, and a jump?

2. Mr. Sears, Mr. Jones, and Mr. Chang are a butcher, a baker, and a candlestick maker, but not in that order. The baker is married to Mr. Sears' sister. Mr. Chang is not the candlestick maker. Mr. Jones is single. Mr. Sears is a regular customer of the butcher. Who has what job?

3. A gadget and a widget equal a boggle. A gadget equals a widget and a smudge. Two boggles equal three smudges. How many widgets equal a gadget?

ANSWERS

(1 pt.) 1. _____

(2 pt.) 2.   Mr. Sears is the _____

Mr. Jones is the _____

Mr. Chang is the _____

(3 pt.) 3. _____

Freshman Meet 1 – SOLUTIONS

Round 3: Logic Problems

1. One jump is two hops. One hop is four skips. How many skips are in a hop, skip, and a jump?

Solution:

$$1 \text{ hop} = 4 \text{ skips}$$

$$1 \text{ jump} = 2 \text{ hops} = 8 \text{ skips}$$

$$1 \quad \text{hop} + 1 \text{ jump} + 1 \text{ skip} = 4 \text{ skips} + 8 \text{ skips} + 1 \text{ skip} = 13$$

2. Mr. Sears, Mr. Jones, and Mr. Chang are a butcher, a baker, and a candlestick maker, but not in that order. The baker is married to Mr. Sears' sister. Mr. Chang is not the candlestick maker. Mr. Jones is single. Mr. Sears is a regular customer of the butcher. Who has what job?

Solution:

	Bu	Ba	Ca
S	X	X	☺
J	☺	X	X
C	X	☺	X

Sears – Candlestick Maker

Jones – Butcher

Chang – Baker

3. A gadget and a widget equal a boggle. A gadget equals a widget and a smudge. Two boggles equal three smudges. How many widgets equal a gadget?

Solution:

$$g + w = b$$
$$g = w + s$$
$$2b = 3s$$

$$2g + 2w = 2b$$
$$2g + 2w = 3s$$
$$2g + 2w = 3(g - w)$$
$$2g + 2w = 3g - 3w$$
$$5w = g$$

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Round 4: Ratio, Proportion, and Variation

All answers must be in simplest exact form in the answer section

NO CALCULATOR ALLOWED

1. Machine X produces paper clips at a constant rate of 20 paper clips per hour. Machine Y produces 10 more paper clips per hour than machine X. Working together, how many hours will it take X and Y to produce 3000 paper clips?

2. If $\frac{a}{b} = \frac{c}{d}$, and $a - b = 72$, find the value of b when $c = 13$ and $d = 5$.

3. x varies directly with y and inversely with z^2. When $x = 10, y = 15$, and $z = 5$. Find the value of x when $y = 5$ and $z = 10$.

ANSWERS

(1 pt.) 1. _____

(2 pt.) 2. _____

(3 pt.) 3. _____

Freshman Meet 1 – SOLUTIONS

Round 4: Ratio, Proportion, and Variation

1. Machine X produces paper clips at a constant rate of 20 paper clips per hour. Machine Y produces 10 more paper clips per hour than machine X. Working together, how many hours will it take X and Y to produce 3000 paper clips?

Solution:

$$x = 20 \text{ clips/hour}$$

$$y = 30 \text{ clips/hour}$$

$$x + y = 50 \text{ clips/hour}$$

$$\frac{3000 \ clips}{50 \ clips/hour} = 60$$

2. If $\frac{a}{b} = \frac{c}{d}$, and $a - b = 72$, find the value of b when $c = 13$ and $d = 5$.

Solution:

$$b = a - 72$$

$$\frac{a}{b} = \frac{c}{d} \rightarrow \frac{a}{a - 72} = \frac{13}{5} \rightarrow -8a = -936 \rightarrow a = 117$$

$$b = 117 - 72 = 45$$

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3. x varies directly with y and inversely with z^2. When x = 10, y = 15, and z = 5. Find the value of x when y = 5 and z = 10.

Solution:

$$\frac{xz^2}{y} = k$$

$$\left[\frac{10 * 5^2}{15} = \frac{x * 10^2}{5}\right] 15$$

$$10 * 5^2 = 3x * 10^2$$

$$5^2 = 30x$$

$$\frac{25}{30} = x \rightarrow x = \frac{5}{6} \text{ or } 0.8\overline{3}$$

WORCESTER COUNTY MATHEMATICS LEAGUE

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TEAM ROUND

All answers must be in simplest exact form in the answer section. (2 points each)

NO CALCULATOR ALLOWED

1. Solve for x:

$$\frac{3}{4}x + \frac{5}{9}x = \frac{2x-3}{6} - \frac{3-2x}{4}$$

2. How many ways can a clerk give a customer $0.25 in change?

3. If $f(x) = 3x - 2$ and $g(x) = |10 - x|$, what is the value of $g(f(4))$?

4. Find two numbers whose sum is 1007 and whose difference is 131.

5. Find the average, as a base 5 number, of 111_2 and 111_3.

6. Thomas, Richard, and Harold each randomly choose one card from a standard deck of playing cards. What is the probability that all three cards chosen will be black? Express your answer as a reduced fraction.

7. Evaluate

$$\frac{6+3-5}{6-3+5} + \frac{7-2(4)+6}{7+\frac{2}{4}-6}$$

8. A varies directly as the product of B and C but inversely as D. If A=100 when B=5, C=2, and D=10, find B if A=500, C=20, and D=3.

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TEAM ROUND ANSWER SHEET

1. _____

2. _____

3. _____

4. _____ and _____

5. _____ (base 5)

6. _____

7. _____

8. _____

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TEAM ROUND - SOLUTIONS

1. Solve for x:

$$\frac{3}{4}x + \frac{5}{9}x = \frac{2x-3}{6} - \frac{3-2x}{4}$$

Solution:

$$\frac{3}{4}x + \frac{5}{9}x = \frac{2x-3}{6} - \frac{3-2x}{4}$$

$$27x + 20x = 6(2x - 3) - 9(3 - 2x)$$

$$47x = 12x - 18 - 27 + 18x$$

$$47x = 30x - 45$$

$$17x = -45$$

$$x = -\frac{45}{17} \; or - 2\frac{11}{17}$$

2. How many ways can a clerk give a customer $0.25 in change?

Solution:

Q	D	N	P
1	0	0	0
0	2	1	0
0	2	0	5
0	1	2	5
0	1	1	10
0	1	0	15
0	1	3	0
0	0	5	0

5. Find the average, as a base 5 number, of 111_2 and 111_3.

Solution:

$$111_2 = 7_{10} \text{ and } 111_3 = 13_{10}$$

The average of 7 and 13 is 10, which is 20_5

6. Thomas, Richard, and Harold each randomly choose one card from a standard deck of playing cards. What is the probability that all three cards chosen will be black? Express your answer as a reduced fraction.

Solution:

$$\frac{26}{52} * \frac{25}{51} * \frac{24}{50} = \frac{2}{17}$$

7. Evaluate

$$\frac{6 + 3 - 5}{6 - 3 + 5} + \frac{7 - 2(4) + 6}{7 + \frac{2}{4} - 6}$$

Solution:

$$\frac{4}{8} + \frac{5}{\frac{3}{2}} = \frac{1}{2} + \frac{10}{3} = \frac{3}{6} + \frac{20}{6} = \frac{23}{6} \text{ or } 3\frac{5}{6} \text{ or } 3.8\overline{3}$$

8. A varies directly as the product of B and C but inversely as D. If A=100 when B=5, C=2, and D=10, find B if A=500, C=20, and D=3.

Solution:

$$\frac{AD}{BC} = K \rightarrow \frac{100 * 10}{5 * 2} = \frac{500 * 3}{B(20)} \rightarrow B = \frac{3}{4} \text{ or } 0.75$$

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Answer Key

Round 1:

1. 20 (Douglas)

2. $\frac{5}{22}$ or $0.2\overline{27}$ (Westborough)

3. 6 (Hopedale)

Round 2:
1. 8 (Bancroft)
2. $-\frac{104}{25}$ or $-4\frac{4}{25}$ or -4.16 (Algonquin)
3. 8 (Assabet Valley)

Round 3:
1. 13 (Bartlett)
2. Mr. Sears is the candlestick maker. Mr. Jones is the butcher. Mr. Chang is the baker. (Assabet Valley)
3. 5 (Douglas)

Round 4:
1. 60 (Quaboag)
2. 45 (Doherty)
3. $\frac{5}{6}$ or $0.8\overline{3}$ (Bromfield)

Team Round:

1. $-\frac{45}{17}$ or $-2\frac{11}{17}$ (Algonquin)	5. 20 (Notre Dame)
2. 13 (Hopedale)	6. $\frac{2}{17}$ (Bartlett)
3. 0 (Quaboag)	7. $\frac{23}{6}$ or $3\frac{5}{6}$ or $3.8\overline{3}$ (Notre Dame)
4. 569 and 438 (Bromfield)	8. $\frac{3}{4}$ or 0.75 (Assabet Valley)